Econ 100B: Nonuniform Pricing Selected Solutions

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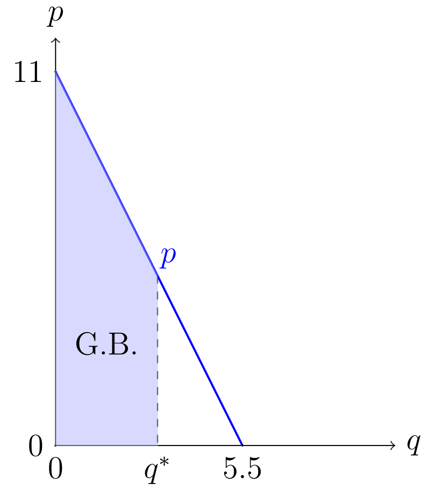
August 29, 2016

9. I run a burger joint. My marginal cost per burger is \$1, and all of my customers share exactly the same inverse demand function, which is $p = 11 - 2x$, where p is the price and x is the number of burgers they consume. I realize that instead of charging a uniform price for each burger, I can increase my profits by offering each consumer an all-or-nothing deal. Which option brings me the most profit?



The firms profit function is given by $\Pi = p - TC$. Hence, the firm will charge a price, p such that he captures the entire gross benefit of the consumer. All that we need to find is the quantity that maximizes the firms profits.

$$\Rightarrow \Pi = \frac{1}{2}(11 + 11 - 2x) \cdot x - TC$$
$$\Rightarrow \frac{d\Pi}{dx} = 11 - 2x - 1 = 0$$
$$\Leftrightarrow \boxed{x^* = 5}$$
$$\Rightarrow \boxed{p^* = 30}$$

11. Blair owns a local business that provides email updates on surf conditions. He is the only supplier of these email updates in Santa Barbara, which gives him a monopoly. The marginal cost of producing another update is zero (and well ignore fixed costs). The inverse demand for these updates in Santa Barbara is $p = 60 - 5q$.

a) Find the profit-maximizing price and quantity, as well as Blairs profits.

$$\begin{aligned}
\Pi &= p \cdot q - TC \\
\Leftrightarrow \Pi &= 60q - 5q^2 - TC \\
\Rightarrow \frac{d\Pi}{dq} &= 60 - 10q = 0 \\
\Leftrightarrow &\boxed{q^* = 6} \\
\Rightarrow &\boxed{p^* = 30}
\end{aligned}$$

b) Now suppose that Blair wants to sell his updates in Goleta as well. The inverse demand in Goleta is $p = 12 - q$. If Blair were to charge a uniform price in both SB and Goleta, and set it so that people in both cities buy his updates, what is the optimal price for him to charge? How much does he sell and what are his profits?

$$q^{AD} = \begin{cases} 24 - \frac{6}{5}p & 0 \leq p \leq 12 \\ 12 - \frac{1}{5}p & 12 \leq p \leq 60 \end{cases}$$

We want to selll in both markets so we only need to find the profit maximizing price for the first region.

$$\begin{aligned}
\Pi &= 24p - \frac{6}{5}p^2 \\
\Rightarrow \frac{d\Pi}{dp} &= 24 - \frac{12}{5}p = 0 \\
\Leftrightarrow &\boxed{p^* = 10} \\
\Rightarrow &\boxed{q^* = 12} \\
\Rightarrow &\boxed{\Pi = 120}
\end{aligned}$$

c) What is the uniform price that earns Blair the greatest total profits (from SB and Goleta combined)? What are his profits?

We only care about maximizing our profits, not which market we sell in so we must check the second region of our aggregate demand function.

$$\begin{aligned}
\Pi &= 12p - \frac{1}{5}p^2 \\
\Rightarrow \frac{d\Pi}{dp} &= 12 - \frac{2}{5}p = 0 \\
\Leftrightarrow &\boxed{p^* = 30} \\
\Rightarrow &\boxed{q^* = 6} \\
\Rightarrow &\boxed{\Pi^* = 180}
\end{aligned}$$

d) Suppose Blair charges different prices in SB and Goleta. For each city, find the profit maximizing price and quantity. What are Blairs total profits?

From part a and c, we know that the profit maximizing price and quantity for the Santa Barbara market are $(p_s, q_s) = (30, 6)$. All that is left to find is the profit maximizing price and quantity for the Goleta market.

$$\begin{aligned} \Pi &= 12q - q^2 \\ \Rightarrow \frac{d\Pi}{dq} &= 12 - 2q = 0 \\ \Leftrightarrow &\boxed{q_g^* = 6} \\ \Rightarrow &\boxed{p_g^* = 6} \end{aligned}$$

Together with the Santa Barbara market, Blaire's total profits from price discriminating will be $\Pi = 180 + 36 = 216$.

15. Stringer Bell sells two products, red candy for \$4 and yellow candy for \$5. In a typical hour, 5 customers will buy only red candy and 4 customers buy only yellow candy, but there are 2 people who buy both. After asking around, Stringer Bell learns that the red-candy buyers would also buy yellow candy if it were to cost \$2, and yellow-candy buyers would also buy red candy if they were sold for \$2. Stringer Bell considers selling red and yellow candy together in a bundle, in addition to selling them separately at the original prices. What is the optimal price of the bundle? (You can assume that the cost of producing a good or creating a bundle is zero).

To solve this problem, we need to first consider each consumer's willingness-to-pay. Two people are already bundling and thus have a willingness-to-pay for the bundle of \$9, the red candy people have a WTP of \$6, and the yellow candy people have a WTP of \$7. Now we can consider each bundle price individually.

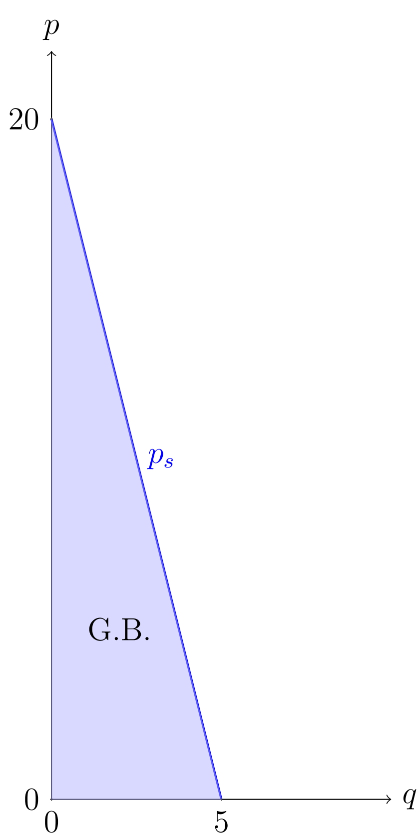
- $p = 6$: Since this price is no greater than the WTP of each customer (later we will discuss an important note), all of them will buy the bundle, which results in a profit of $\Pi = 11 \cdot 6 = 66$.

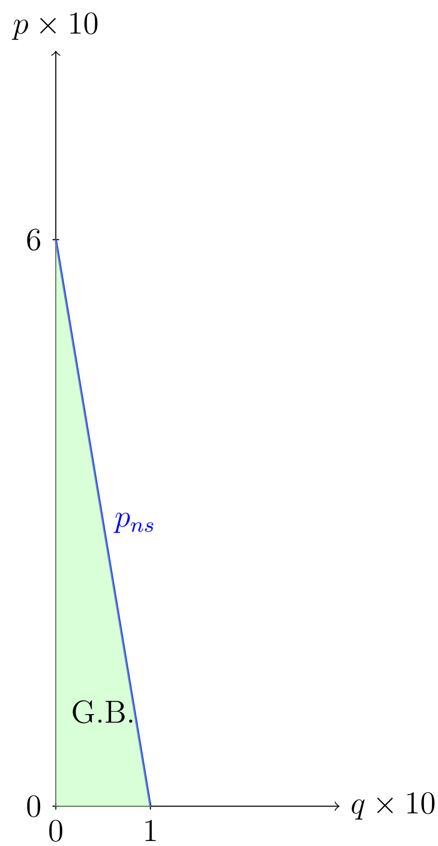
- $p = 7$: At this price, only the original bundle buyers and the yellow candy people will buy the bundle, which results in a profit of $\Pi = 6 \cdot 6 + 5 \cdot 4 = 56$.

- $p = 8 \;\; or 9$: At these prices, only the original bundlers will purchase the bundle, which results in a profit of 56 and 58, respectively.

Clearly, the profit maximizing price is $\boxed{p = 6}$. In this problem, I glossed over a very important point. Recall that, despite not working with utility functions in these problems, consumers are always trying to make themselves as well off as possible (i.e. maximize their utility). What is one way that we measure an increase in well-being from trades? We can consider an economic agent's consumers surplus. In other words, consumers will make their buying decisions in a way that maximizes their consumers surplus. The implications of this may be subtle. In some cases, despite being able to afford the bundle, consumers may still choose to buy the single good because they receive a larger surplus form that purchase. This becomes important in the TV & Internet practice problem.

52. FitYogah offers yoga classes to two consumers and has no marginal cost. One consumer is a student with inverse demand $p_s = 20 - 4q$ and the other is a non-student with inverse demand $p_{ns} = 60 - 6q$. FitYogah can not tell the difference between the two customers so it offers a small package of 5 classes targeted to the student and a large package of 10 classes for the non-student. What price should FitYogah charge for each package to maximize its profits and make sure that each customer is willing to buy the intended package?





Because the student's gross benefit is smaller than the non-student's gross benefit, we can charge the full benefit for the student.

$$p_5 = \frac{1}{2} \cdot 20 \cdot 5$$

$$\Leftrightarrow \boxed{p_5 = 50}$$

Now, we need to charge a price for the 10 class package that makes the non-student perfectly indifferent between the two packages. This price captures as much of his benefit as possible without making him buy the 5 class package.

$$\frac{1}{2}(60+60-6\cdot 5)\cdot 5-p_5=\frac{1}{2}\cdot 60\cdot 10-p_{10}$$

$$\Leftrightarrow 225-50=300-p_{10}$$

$$\Leftrightarrow \boxed{p_{10}=125}$$