E. Charlie Nusbaum
Econ 204A
December 1, 2017

Week 7 Section Notes

Problem 4.9. Consider an overlapping generations economy where individuals in generation t maximize utility

$$U = \ln(c_{1t}) + \beta \ln(c_{2t+1})$$

They work one unit when young, earn a wage w_t, save an amount a_t, and earn interest r_{t+1} on their savings. Ouput is produced with a Cobb-Douglas technology $Y_t = K_t^{\alpha} L_t^{1-\alpha}$ and there is 100% depreciation. The number of individuals in generation t is L_t. The size L_t of generations t grows at a fixed rate $n > 0$. The governmnet operates a pay-as-you go social security system as follows: Each period, the young pay a tax $T_{1t} = \tau w_t$, where $0 < \tau < 1$ (since labor supply is fixed, the tax is lump sum). The receipts are given to the old as a transfer $(-T_{2t})$.

a) Set up the individual optimization problem for generation t, derive the first order conditions, and solve for the asset accumulation as a function of T_{1t} and T_{2t+1}.

b) Explain why $\frac{-T_{2t+1}}{w_{t+1}} = \tau(1+n)$. Show ho next period's capital-labor raio relates to the current capital-labor ration. [Hint: simplify $\frac{w_{t+1}}{1+r_{t+1}}$]

c) Determine the steady state cpaital-labor ratio k^*. Show that k^* is decrease in τ.

Problem 4.10. Consider an overlapping generations economy with log-utility and Cobb-Douglas production. Individuals in generation t maximize utility

$$U = \ln(C_{1t}) + \beta \ln(C_{2t+1})$$

Individuals earn a wage w_t, save an amount a_t, and earn interest r_{t+1} on their savings. Output is $Y_t = K_t^{\alpha} L_t^{1-\alpha}$. Capital fully depreciates after one period. Generation t has L_t members. Population grows at a fixed rate n. The government may impose lum-sum taxes T_{1t} on the young and T_{2t} on the old. The government budget equation

$$D_{t+1} = (1 + r_t)D_t - [L_t T_{1t} + L_{t-1} T_{2t}]$$

describes the dynamics of government debt.

a) Set up the individual optimization problem for generation t, derive the first order condition, and solve for a_t as a function of w_t, r_{t+1}, T_{1t}, and T_{2t+1}.

b) Suppose the government is inactive so $T_{1t} = T_{2t} = D_t = 0 \, \forall t$. Show how next period's capital-labor ratio depends on the current value of k_t. Show that the capital-labor ratio converges to a steady state k^*. Under what conditions about β is the economy dynamically inefficient?

c) Suppose government debt carries into the next period that is proportional to the size of the yound generation, $D_{t+1} = d \cdot L_t$, for all t. where $d > 0$. Suppose $T_{2t} = 0$ and T_{1t} is set so that the budget equation is satisfied. Show how k_{t+1} depends on k_t and d. Derive k^*. Show that a higher d implies a lower k^*.