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Due: *April 13, 2018*
Econ 204C

Problem Set 1

Problem 1: The Hodrick-Prescott Filter

The Hodrick-Prescott filter is a widely used technique for decomposing a time series into "trend" and "cyclical" components. Let $\{y_t\}_{t=1}^{\infty}$ be a given series. (All time series needed for this problem set are available on my webpage. All data is in 2009 U.S. dollars.) We think of y_t as the sum of a growth component, z_t, and a cyclical component, $y_t - z_t$. Let λ be a parameter to be specified later, and consider the problem

$$\min_{z_1, z_2, \ldots, z_T} \sum_{t=1}^{T} (y_t - z_t)^2 + \lambda \sum_{t=2}^{T-1} [(z_{t+1} - z_t) - (z_t - z_{t-1})]^2$$

a) The first order conditions for the above problem are linear, which is to say they can be written in the form

$$Pz = y$$

Find P.

b) Using your favorite numerical tool, write a program that seperates any time series into a growth and a cyclical component given λ.

c) Let $\{y_t\}$ be annual U.S. real GDP, 1929-2014, in logarithms. Let $\lambda = 400$. Using the program written in (b) seperate $\{y_t\}$ into growth and cyclical components. Plot the series and the growth component in one graph and the cyclical component in a different graph.

d) Repeat the exercise in (c) but using $\lambda = 0$ and $\lambda = 100,000$. Explain the effect of increasing the value of λ. How does the HP filter compare to a linear regression when $\lambda \to \infty$?

e) Experiment with different λ values until you get the best decomposition into growth and cyclical components. What do you mean by "best?"

Problem 2: Optimal Growth with Linear Utility

Consider the optimal growth model with linear utility, Cobb-Douglas production and 100% depreciation every period. The social planner maximizes

$$\sum_{t=0}^{\infty} \beta^t c_t$$

subject to the constraints:

$$\begin{aligned}
k_{t+1} &= Ak_t^{\alpha} - c_t \\
c_t &\geq 0 \\
k_t &\geq 0 \\
k_0 &> 0
\end{aligned}$$

As always, assume $\alpha \in (0, 1)$, and $A > 0$.

a) Prove that there exists a solution to this optimal growth problem. You can do this by finding any finite number which is bigger than the highest possible utility level.

b) Describe as best you can the solution to this planner's problem. You should note that mechanical application of first-order conditions and the Euler equation is unlikely to find the solution.

c) Suppose instead that $\alpha = 1$, so that we have an "AK" model. Identify the conditions under which there is a solution to the planner's problem, and describe the solution under these conditions.

Problem 3: Growth with Technological Progress

Consider the optimal growth model with per-period utility defined by $U(C) = \frac{C^\gamma}{\gamma}$. Production technology is given by a constant returns to scale function, $F(K, L)$, which takes capital and effective labor units as inputs. Capital depreciates at rate $\delta > 0$ and households discount the future at by a factor $0 < \beta < 1$. Initial capital, K_0, and effective labor, L_0, are given. Thus, the sequential problem is given by

$$\max_{(C_t, K_{t+1}, I_t)_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^\gamma}{\gamma}$$

subject to

$$\begin{aligned}
&C_t + I_t \leq F(K_t, L_t) \\
&K_{t+1} = (1-\delta)K_t + I_t \\
&K_0, L_0 \geq 0 \ \textit{given}
\end{aligned}$$

a) Suppose that effective labor, L, does not grow. Write the optimal growth problem recursively.

b) Suppose instead that effective labor, L, grows by a factor $(1 + \lambda)$ each period. That is that there is an additional constraint given by $L_{t+1} = (1 + \lambda)L_t$. Renormalize the problem from (a) to make the model stationary (i.e. transform to efficiency units). Write the renormalized problem recursively and show how this problem is related to the problem from part (a).