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Econ 204B
Due February 7, 2018

Problem Set 2

Problem 1. *Connection between log and power utility.* Consider a CRRA utility function of the form

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$$

(a) Prove that $\lim_{\gamma \to 1} u(c) = \ln(c)$

(b) The Arrow-Pratt measure of relative risk aversion is given by

$$\gamma(c) \equiv \frac{-cu''(c)}{u'(c)}$$

The elasticity of substitution between consumption at two points in time t and s is given by

$$\sigma(c_t) \equiv \frac{u'(c_s)/u'(c_t)}{c_s/c_t} \cdot \frac{\mathrm{d}(c_s/c_t)}{\mathrm{d}(u'(c_s)/u'(c_t))}$$

Show that $\lim_{s \to t} \sigma(c_t) = \frac{1}{\gamma(c_t)}$. [Hint: Let $c_s = x \cdot c_t$ to start].

Problem 2. *Value Function Iteration.* Consider a model economy populated by a unit measure of households that solve the following problem:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}-1}{1-\gamma} \qquad s.t. \qquad c_t + a_{t+1} = (1+r)a_t + w, \qquad c_t, a_t \geq 0$$

(a) Write the sequential problem as a recursive problem. Be sure to identify what the state and choice variables are for the household.

(b) Assume that equilibrium prices are given by $(r, w) = (0.04, 2.09)$, and that $\beta = 0.98$ and $\gamma = 2$. Perform value function iteration to solve for the value function and policy function (numerically) using both an initial guess for $V_0(a) = 0$ and $V_0(a) = sin(a)$. Graph both of these initial value functions over a grid of possible $a's$

(c) Write code that performs value function iteration and graph the value function at iterations 10, 50, 100, 500, and 1000 all on the same graph. Plot the policy function on a seperate graph for the converged solution.

(d) Now assume that time is finite and ends after period $T = 20$. Again solve for the optimal value and policy functions. Plot the policy function as a function of age for several different current asset holdings. [Note: Do not use a shooting algorithm to solve this problem.]

(e) Assume households begin life with 0 assets. Simulate the life-cycle savings behavior of the representative agent. Plot your results.