

### Econ 10A: Problem Set 5

- (1) A security pays \$100 at the end of each of years 1, 2, 3, and 4 and the annual interest rate is  $r$ .
- What is the present discounted value of this security? (Hint: You may not be able to calculate a numeric value.)
  - Does the present discounted value increase or decrease when the interest rate increases? Explain your answer in 2 sentences or less.
- (2) Pirates are about to set sail on a 2-period journey (trip). They have 100 bags of barley (food) and must decide how much to consume in period 1 ( $c_1$ ) and period 2 ( $c_2$ ). The pirates get all the barley in period 1 and none in period 2. Unfortunately, rats will eat 50% of any barley saved to consume in period 2. Assume the Captain, who rations the food, has the following utility function:  $U(c_1, c_2) = c_1c_2$ .
- What is the “real interest rate?” How much does the Captain choose to consume in each period? (Hint: The “price” of barley in each period can be assumed to be 1.)
  - Now assume the pirates attack another ship in period 2, stealing 300 bags of barley. How many bags would this stolen barley have been in period 1 (assuming all ships have rats)? Assume the pirates could borrow against future barley spoils. What is the intertemporal budget constraint in this case?
  - Now, in addition, assume that the pirates have a garden on their ship. Whatever barley the Captain chooses to plant will grow into 1.5 times as much barley in period 2. Planted barley is also subject to attack by rats. What is the “real interest rate?” What levels of consumption does the Captain choose in each period?
- (3) Ralph’s utility function is the square root of his consumption:  $U(c) = \sqrt{c}$ . He currently works and receives a weekly salary of  $m = \$900$ . There is a 20% chance that he will be get the Floor Blues Flu, which requires him to stay home this week and receive no income. Ralph has the option to purchase  $K$  dollars of insurance ( $0 \leq K \leq m$ ), where each dollar of insurance costs \$0.20.
- What is Ralph’s consumption if he gets sick in the case where he has bought some insurance? (Denote his consumption in the state of the world with sickness by  $c_b$ )
  - What is Ralph’s consumption if he does not get sick in the case where he has bought some insurance? (Denote his consumption in the state of the world without sickness by  $c_g$ )
  - Draw Ralph’s contingent budget constraint on a graph with  $c_b$  on the x-axis and  $c_g$  on the y-axis. Make sure to fully label your graph.
  - How much will  $K$  be in order to maximize expected utility?
  - How would his choice of  $K$  change if his utility function was  $U(c) = c^2$ ?